In Praise of Fostering Anarchy in Research and Teaching

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In this paper I attempt to make a case for anarchy in research against the current practice of picking winners in universities at advanced levels of education and research. By considering a paradigmatic example of freedom in speculative intellectual activities leading to unintended consequences of enormous benefit to mankind, I try to substantiate a case for this. The example I consider is the way issues in the foundations of mathematics paved the way for what came to be known as the IT revolution. It is a counter-factual narrative and may – hopefully, will – provide an antidote to the current orthodoxy's regimented non-vision of "picking winners", ex ante, without any historical substantiation.

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Preamble

I want to begin this paper[1, 2, 3] anecdotally, to give it the flavour I want it to have, of an informal lecture to beginning graduate students, on the attitudes one should have towards research. I shall, therefore, preface it by the story of the kind of approach to research that was fostered by my own Cambridge maestro, Richard Goodwin, as reported, with first-hand experience, by one of his own most illustrious students, the Nobel Laureate Robert Solow:

> There was something more important, however. It is clear in my mind that when I asked what I must have thought were devilishly clever or profound questions, Mr Goodwin did not take a high and mighty – and defensive – line. His answers made it plain that he had plenty of sceptical doubts of his own. I may be inventing this, but I seem to recall that he sometimes suggested that, well, one could not actually believe this or that, but it was an ingenious line of thought, perhaps worth following just to see where it came out. One could always reject it later, and then one would have a better idea of what one was rejecting. If that actually happened, then I was getting my introduction to the theorist's frame of mind...
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> …I continued to learn from [Goodwin], both in the substance of economic theory …. and in a more subtle way that I do not know how to describe except as a matter of intellectual style. *The unspoken language was that if a thing is worth doing it is worth doing playfully. Do not misunderstand me: 'playful' does not mean 'frivolous' or 'unserious'. It means, rather, that one should follow a trail the way a puppy does, sniffing the ground, wagging one's tail, and barking a lot, because it smells interesting and it would be fun to see where it goes.*
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> (Solow 1990: 32-33; italics added).

I am convinced that this spirit of *enlightened "playfulness" in research* is what animated Turing – and, of course, Solow – and it is exactly this that is being discouraged in the current intellectual environments of research institutions, plagued and harassed as they are with pressures to produce results that are measurable in the marketplace, *as if ideas can be produced without speculations, failures and traumas*.

Moreover, even while espousing the virtues of globalisation of the market for goods and services in the conventional sense, academic institutions – in particular – are increasingly parochial in the way they are administered,[4] hoping to outline strategies to "pick winners", nationally, and out-compete other nations in the so-called marketplace for ideas and their immediate application for monetary rewards. Crass cost-benefit analysis, without the slightest understanding of the kind of assumptions required for such methods to make serious sense, motivates evaluations and ordinary promotions.

My two other prefatory remarks refer to celebrated but, mercifully, falsified prophetic pontifications by two almost saintly intellectual giants of the 19th century: Lord Kelvin and John Stuart Mill. The former is reputed to have suggested, on the eve of the works by Planck and Einstein that changed the intellectual map of the natural scientist as a physicist, that *all the problems of physics had been solved:*[5] "except for just two anomalies: that of the Michelson-Morley experiment, on the one hand, and Black Body radiation on the other"! The one led to the relativistic revolution; the other to the quantum intellectual cataclysms.

As for the great and saintly John Stuart Mill, in what can only be called an unfortunate moment of weakness, he etched for posterity these (in)-famously un-prophetic thoughts on the "end of the theory of value":

> *Happily, there is nothing in the laws of Value which remains for the present writer to clear up; the theory of the subject is complete*: the only difficulty to be overcome is that of so stating it as to solve by anticipation the chief perplexities which occur in applying it: and to do this, some minuteness of exposition, and considerable demands on the patience of the reader, are inevitable.
> (John Stuart Mill (1848, [1898]), Bk III, Ch 1, p 266; italics added.)

These words were coined on the eve of Marx's great and revolutionary works and not many years before the even more significant marginal revolutions in value theory.

Research and intellectual adventures can and must always be open-ended and the institutions that underpin open-ended research have, themselves, to be founded on structures with flexibilities – rather like the way Herbert Simon advocated organisations to be semi-decomposable so as to facilitate evolution.

The task I am setting myself is to outline the path, from a Whig perspective, of outlining the origins and tracing the path taken by an idea, or a set of ideas, founded on rigorous speculations in the pure sciences, philosophy and metamathematics, and leading to unforeseen developments in the applied sciences, even leading to unimagined policy ramifications for the "real" world. The narrative is, inevitably and to some extent regrettably, in a Whig mode. Since I aim to start from the "germ of an idea", it is also, invariably, a celebration of the "lone" scientist[6] and his or her adventures through the maze that is the intellectual heritage of the times, "standing on the shoulders of the many giants" who forged the traditions and concepts that are the initial conditions for any such journey.

In taking this line of approach to outline the morale of my narrative, I must, of necessity distort the picture of actual practice and real history – even of the purest of the pure sciences, say number theory or aspects of mathematical analysis, especially during the period of relevance for my story – of the 20th century. In this period, the growth of "corporate" science, with ramifications even for research in the purest of the pure sciences, has been remarkable. In the later part of the 20th century, to this has been added, also, the burgeoning role of the venture capitalist, within emerging forms of organisations. These institutional aspects are obviously part of a complete picture, but I shall, nevertheless, celebrate the germination, nurturing, growth to maturity and emergence in the form of a fully-fledged theory, *of an idea in the mind of a driven individual*, within the settings of an intellectual culture and tradition that is open to all.

I doubt anyone would dispute my claim that two of the most edifying examples of such cases, for the 20th century, are Alan Turing and Andrew Wiles.[7] But the work of the former has, within one lifetime, manifested itself in a "Whig justification", as far as the applied scientific ramifications are considered. The conceptual innovations that culminated in the resolution of Fermat's Last Theorem at the hands of Andrew Wiles will, surely, have its applied scientific ramification – but such "unintended consequences" remain for the unforeseeable future.

An Adventure of Ideas

In 1928 the first International Congress of Mathematicians since the first world war – since, in fact, 1912 – was held in the Italian city of Bologna. Since that tragic war[8] German mathematicians had not been invited for international meetings. Italians, in those heady pre-Fascist and pre-Nazi days, were determined to make the occasion in Bologna truly international and invited the Germans. Despite opposition by some leading mathematicians in Germany, the great Göttingen successor of the mathematical spirit of Gauss, Riemann and Klein, for a time the only rival to Henri Ponicarè as a truly universal mathematician, David Hilbert, led a delegation of 67 German mathematicians to the Bologna congress.

In 1925 and 1927 Hilbert had begun to crystallise his programme for the foundations of mathematics in a system which came to be called *Formalism*, in contrast to two other systems, that of *Intuitionism*, almost single-handedly pursued and promoted by the Dutch mathematician Luitzen Brouwer, but not without support from leading French mathematicians for some aspects of its epistemology, and what was called *Logicism*, a system that was essentially the outcome of the message of the monumental programme to reduce mathematics to logic that was represented in the great three-volume work by Russell and Whitehead. These systems of mathematical thought were about the deepest issues in the methodology, philosophy and epistemology[9] of mathematical truth, the nature of the objects of mathematical investigation and the language and logic of the language to be used in talking about mathematics – i e, metamathematics.

Partly as a result of the so-called *antinomies* of set theory – one of the most celebrated of which was Russell's Paradox of the "set of all sets that do not contain themselves as members" – mathematicians at the turn of the 19th century to the 20th had begun to be more circumspect of arbitrary definitions and untrammelled methods of proof. Hilbert, notwithstanding the known antinomies and the dangers of unconstrained methods of proof, particularly in proving the existence of a mathematical object as a consequence of not being able to derive a contradiction in the defining criteria, had seemed to promote the idea of mathematical formalism as a symbol manipulation game, with its own rules without any discipline on the nature, contents and structure of thought.

Brouwer, at a kind of polar opposite end, underpinned by his belief, not negligibly influenced by a philosophical mysticism and, ex post, something like Husserlian Phenomenology, that mathematical objects were the autonomous creations of the human mind, endeavoured to discipline the allowable techniques of demonstrating the existence of mathematical objects and their definitions.

The demonstration of the existence of a mathematical object – say even an abstract one such as the equilibrium price configuration of an economy, the prices at which market supply equals market demand – should be accomplished by *constructive methods of* *proof*; i e, methods that could, in principle, be used by an "engineer" actually to construct such an object with ruler, compass, chisel, lathe and so on. Thus, to say that a mathematical object *exists* if the decimal representation of π, say, contains a particular sequence of 9's at a particular place in the expansion, is to say nothing and the formalist mathematician to retort that even if s/he does not know whether such a statement is true of the object π, *God will know,* would bring forth the retorts from Brouwer that he did not have a pipeline to God and if God had mathematics to do, he can do it himself; man's mathematics was not necessarily that of God's.

In other words, Brouwer and the Intuitionists would restrict the allowable methods of proof for mathematicians to those that did not appeal to untrammelled infinities, undecidable disjunctions and so on – almost banning magic and metaphysics from mathematical practice, as if telling Newton not to dabble in alchemy if we are to believe his mechanics and optics!

To which Hilbert (would) reply: "With your [Brouwer's] methods, most of the results of modern mathematics would have to be abandoned, and to me the important thing is not to get fewer results but to get more results".

God, *Golem*, the *infinite*, the *infinitesimal* and many other omniscient beings were routinely invoked in these metamathematical debates on the foundations of mathematics in an exhilarating atmosphere encouraging speculative thought, *without the slightest concern for practical applications*, immediate or distant.

By the time of the Bologna meetings of the International Congress of Mathematicians, Hilbert had given two lectures[10] building towards a final crystallisation of his position, such that when formulated as challenges to mathematicians in the form of well-posed problems, and answers given, then debate would forever be silenced and mathematicians would be allowed to go on with their normal activities, untrammelled by any kind of constrains by a thought-police of any sort, however enlightened in method, epistemology or philosophy. Hilbert had stated his credo, not only by his outstanding mathematical works as examples of the philosophy he was advocating – as, indeed, was the case with Brouwer[11] – but also by explicitly stating in his influential address to the Paris International Congress of Mathematicians in August, 1900, titled famously and simply: "Mathematical Problems" (Hilbert 1900 [1902], p 444, italics in the original):

> [T]he conviction (which every mathematician shares, but which no one has as yet supported by a proof) that every definite mathematical problem must necessarily be susceptible of an exact settlement, either in the form of an actual answer to the question asked, or by the proof of the impossibility of its solution and therewith the necessity failure of all attempts…
> Is this axiom of the solvability of every problem a peculiarity characteristic of mathematical thought alone, or is it possibly a general law inherent in the nature of the mind, that all questions which it asks must be answerable? For in other sciences also one meets old problems which have been settled in a manner most satisfactory and most useful to science by the proof of their impossibility….
> This conviction of the solvability of every mathematical problem is a powerful incentive to the worker. We hear within us the perpetual call: There is the problem. Seek its solution. You can find it by pure reason, for in mathematics there is no *ignoramibus*.

Even as far back as 1900, in that same famous lecture, Hilbert had also stated,[12] clearly and unambiguously, the acceptable criteria for the "solution of a mathematical problem":

> [I]t shall be possible to establish the correctness of the solution by means of a finite number of steps based upon a finite number of hypotheses which are implied in the statement of the problem and which must always be exactly formulated. This requirement of logical deduction by means of a finite number of processes is simply the requirement of rigour in reasoning.

These were the methodological and epistemological backdrops against which, in Bologna in 1928, Hilbert threw down the gauntlet to his foundational detractors, in the clear conviction that the answers to the questions he was posing would be answered – surely, also, to substantiate his own philosophy of mathematics:

Is mathematics *complete* – in the sense that every mathematical statement could be rigorously – rigour interpreted in the above finitary sense – proved or disproved;

Is mathematics *consistent* – in the sense that it should not be possible to derive, by valid proof procedures, again in the sense of finitary rigorous proof stated above, universally false mathematical statements within a formal mathematical system;

Is mathematics *decidable* – in the sense of using a definite finitary method, it was possible to demonstrate the truth – or falsity, as the case may be – of a mathematical assertion.

Hilbert was born in Wehlau, near what was then called Königsberg[13] the capital of east Prussia. On 8 September 1930 Hilbert gave the opening address to the German Society of Scientists and Physicians, in Königsberg,[14] titled: "Naturkennen und Logik". This lecture ended famously echoing those feelings and beliefs he had expressed in Paris, 30 years earlier (italics added):[15]

> For the mathematician there is no Ignoramibus and, in my opinion, not at all for natural science either. … The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, there is *no* unsolvable problem. In contrast to the foolish Ignoramibus, our credo avers:
> *We must know,*
> *We shall know.*[16]

A day before that, on Sunday, September 7, 1930, at the roundtable discussion on the final day of the Conference on Epistemology of the Exact Sciences, organised by the Gesellschaft für Empirische Philosophie, a Berlin Society allied to the Wiener Kreis, the young Kurt Gödel had presented what came to be called his *First Incompleteness Theorem*. In fact, in one fell swoop, Gödel had shown that it was recursively demonstrable that in the formal system of classical mathematics, assuming it was consistent, there were true but unprovable statements – i e, incompleteness and, almost as a corollary to this famous result, also that mathematics was inconsistent.[17] Two of the pillars on which Hilbert was hoping to justify formalism had been shattered.

There remained the third: *Decidability*. The problem of resolving this question depended on finding an acceptable – to the mathematician, metamathematician and the mathematical philosopher – definition of *definite finitary method*. In one of the celebrated confluences and simultaneous discoveries that the history of science and mathematics seems to be littered with, Alan Turing and Alonzo Church came up with definitions that, ex post, came to be accepted by mathematicians, logicians, etc, as encapsulating the intuitive notion of *definite finitary method*, now routinely referred to as "algorithms".[18]

Once this was done the unadulterated genius of Alan Turing devised, entirely with the aim of answering the question of *decidability* posed by Hilbert, the now celebrated "Turing Machine", the prototype of the modern stored program computer. *Every* single theoretical concept of the architecture of the modern computer, the very laptop on which I am "penning" these speculative thoughts, was rigorously formulated in that path-breaking paper of 1936-37 by Alan Turing (1936-37).

To this story of the inspiration in the purest of pure mathematical speculations of the origin of the architecture and the mathematics of the computer, one can add, first of all, the story of the justification von Neumann gave for the architecture of the stored program computer, in his famous 1945 EDVAC report, in terms of the structure and processes immanent in the neuronal basis of the logical structure of brain activity (McCulloch and Pitts 1943)[19] – the logic of what Sir Charles Sherrington called activities of *the enchanted loom*[20] that is the brain. Then the story of how Claude Shannon was led to suggest how Boolean algebra could be used in the analysis and synthesis of switching and computer circuits. And also the origins of the formal, statistical, basis of information theory developed by Claude Shannon lying in the speculations of the tragic character of Ludwig Botlzmann and the imaginary ideal information processor that the great James Clerk Maxwell devised and we have come to know as "Maxwell's Demon".[21] If all these are considered, then most of the methodological, epistemological and technological underpinnings of what is called the "Information Society" or information technology (IT) and so on will be understood.

Tentative Lessons

The reforms in the nature and scope of universities, encouraged and implemented by Alexander von Humboldt, in the early years of the 19th century – exemplified the concrete example of the founding of Berlin university as a pilot reform university, integrating, organically, teaching and research – became the norm at all the ancient and leading universities of the "western world". Humboldt's enlightened vision of *the indivisible unity of teaching and research*, each strengthening the other, was one that has come under increasing danger in recent decades.

Moreover, and regrettably, the vision was never incorporated as a natural aim in the new and bold initiatives that went into the building of universities in the post-colonial world – even in Latin America, where independence from Spanish dominance came centuries ago.[22] In particular, in south Asia, China and Latin America, the balance between teaching and research, within the scope defined for universities seems to have always been heavily biased in favour of the former. Research was largely confined to dedicated, specialised, departments specifically constructed within existing larger units – such as central banks – or in specialised institutes, such as the various institutes of advanced studies or national institutes of fundamental research, especially in India. However, even these were, at least in their initial enthusiasm, almost entirely devoted to the sciences, more specifically to those sciences with close proximity to the applied and engineering possibilities of national life and prestige (atomic energy, being a prime example).

Contrary to popular expositions and mass market impressions, none of the developments I have outlined above, came about by a priori blueprinting of national research strategies, implemented at universities of national research centres, that were to "pick winners". It was almost entirely the freedom, bordering on the "anarchy of research", fostered in environments that allowed pure speculation on deep issues that have always driven man's curiosity.

The same kind of story can be told also for the origins of the inspiration that led to the Crick-Watson results and the DNA-based, molecular biology-genetic revolution. One can trace a significant inspiration in a lecture given in Dublin by Erwin Schrödinger (1943),[23] many years before Pauling, Crick and Watson began to try to crack the genetic code. Indeed, generalising from these three significant episodes, surely one can tell a similar story for many of the innovations that have changed the path of mankind's destiny.

My thesis is simply a variant of the theme Koestler (1959, [1964]), made famous in his fascinating "History of Man's Changing Vision of the Universe", as the story of the "performance of *Sleepwalkers*" trying to "understand Nature, not conquer it". Koestler's Sleepwalkers, Copernicus, Brahe, Kepler, Galileo, Newton …, did not set out to revolutionise technology but their speculative activities led to that revolution as a by-product. My point is that today's Sleepwalkers, Hilbert, Brouwer, Russell, Whitehead, Sherrington, Gödel, Turing, Shannon, McCulloch, …, and others, were speculating on the foundations of mathematics and the epistemology of the mind, a by-product of which is the much vaunted Information Society. The somnambulant speculative activities of Koestler's Sleepwalkers provided the foundations of the industrial revolution and the technological society. The modern Sleepwalkers did not set out, in their somnambulant speculative activities, to create the modern Information Society and its underpinning in IT; it was, as before, an inadvertent by-product.

The current protocols, to "pick winners" by rewarding results that are potentially only a nose length away from being realised, is a travesty of the message of history.

NOTES

1 The original inspiration to tell the story I am trying to outline in this paper came from three irritating and sad experiences. The first was the actual, forceful, uncompromising advocacy of encouraging young graduate students, at my own institution, to pursue lines of research that do not deviate from orthodoxy, "to fish only in well fished waters", as it was uncompromisingly stated. Second, there is the utterly deplorable pre-occupation to evaluate the performance of colleagues in terms of criteria that deaden any adventures in intellectual exploration. Finally, the third, more specifically intellectual motivation is the crass epistemological attitude, promoted without any of the usual intellectual humilities of frontier research, that "the world is probabilistic" and, hence, all modelling visions must start from assumptions about the probabilistic structure of the external world and its events. The particular story that is outlined here, ultimately, is a direct counter-example to this monomaniacal vision of the external world and its possible structures.

I must add one important caveat to the institutional backdrop against which I try to paint a vision for university teaching and research, one which I mention in slightly greater detail in the concluding section of this essay. It is that the paradigmatic institutional exemplar for my narrative is a traditional "western" university – in which category I also include the older universities of Japan (having been a graduate of Kyoto university). With some experience of university and institutional affiliation in India, Sri Lanka, Mexico and China, I realise that such institutions in south Asia, Latin America and parts of east Asia and, possibly, in other areas of the developing world, never – perhaps till very recently – implemented the "Humboldtian Reforms" for universities. Thus, universities were mainly meant to concentrate on teaching – including at advanced levels, bordering on frontier research – while research was left as the focus of dedicated research institutes.

2 The subtitle is meant to be slightly provocative and not to be taken too literally. It is not that I am advocating "anything goes"; it is more in the spirit of Feyerabend's mischievous title, *Against Method* – against inflexibility in institutional and individual structures and behaviour and, above all, against

the current institutionalised philosophy of "picking winners" on the basis of immediately past successes, measured almost universally by instant applicability. Alan Turing's almost playful research results in the foundations of mathematics, theoretical computer science and morphogenesis revolutionised applicable mathematics, information theory, applied mathematics and much else. In this paper, I take up, as a single case study, just one aspect of this example of the profound "consequences of unintended speculation". A similar study could easily substantiate not only the other three aspects of just Turing's own work, but also the path towards Crick and Watson, the intellectual and institutional backdrop to the genesis of modern quantum mechanics, to name just a few of the most significant issues that changed the visions of modern humankind.

3 If the example I deal with in this essay proves satisfactory, the next step I would take is the outlining of the case histories of the *quantum revolution*, from about Planck, via Bohr, to Heisenberg, Schrödinger, Dirac and Feyerabend and Bell; the path towards Crick and Watson, from Schrödinger and, then, beyond DNA to the *genome sequencing exercises*; and the *adventures in nonlinear dynamics*, from Poincaré and Birkhoff (the elder!) towards Smale and Arnold, via van der Pol, Cartwright-Littlewood and Levinson. Serendipitously, *Alan Turing will turn out to have played a significant role in all of the stories*!

4 The disgraceful example of the so-called Bossi-Fini Law, in Italian universities is the kind of thing I have in mind, explicitly, in this case. But the obverse side of that coin is, of course, the fact that I am, in fact, employed – and largely happily so – at an Italian university, largely animated by enlightened broadmindedness. A recent personal example I had to experience was the obdurate attitude displayed by the administration at my university in the case of a very minor payment to a scholar of international distinction, simply because he did not happen to be an EU national. In the end I had to advance payment to the order of €5000 out of my own meagre pockets, to make sure that promises were honoured.

5 The actual statement, made in an address to an assemblage of physicists at the British Association for the Advancement of Science in 1900, seems to have been: "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." Axel Leijonhufvud claimed, in 1968 that Robert Solow (1965) had echoed a similar sentiment for "short-run macroeconomics". However, the careful reader of the article would note that Leijonhufvud deftly omitted the context and some of the caveats in Solow's guarded statement about "short-run macroeconomic theory" being "well in hand *along exactly the line that Lerner described yesterday*" (ibid: 146; the italicised phrase is, in particular, omitted by Leijonhufvud).

6 The efforts of a group of a few individuals, as in the obvious case of Crick and Watson and the emergence of the DNA breakthrough, is a counter-example to the romantic "lone scientist" vision. But I believe a fuller story, tracing the antecedents of the path towards the Crick-Watson breakthrough, going back even only to the work and hypotheses of, say, Linus Pauling and Erwin Schrödinger, might substantiate the "lone scientist" vision at a deeper level (see, in particular, Olby (1994), for an elegant and exhaustive story of "The Discovery of DNA").

7 The case of Srinivasa Ramanujan should form a part of this example, but to substantiate this almost romantic episode in 20th century intellectual achievement, I will need to outline the impact of pure number theory in applied science. Space does not permit to indulge in this wonderful example of individual dedication – both on Ramanujan's part and Hardy's unreserved determination - but also on the institutional freedom that the college system at Cambridge made possible for a determined scientist of international repute.

8 Incidentally, it was the trauma generated by the meaningless slaughter of a whole generation of mathematicians during that tragic war that led to the philosophy underlying the formation of the influential French group of mathematicians who called themselves the "Bourbakians". Their influence had far-reaching consequences even for mathematical economics and the education of economists at some of the best graduate schools of economics, all over the world, as a result of the influence of Gerard Debreu, who was himself deeply influenced by the Bourbakist's vision of mathematical methodology and structure.

9 In the sense of the interaction between the mathematician's "legitimate" *methods* for gaining knowledge about the world of mathematical objects; the "acceptable" notions of what such knowledge should look like; and the mathematician's conceptions of the world of mathematical objects. Thus there were unadulterated *Platonist* mathematicians who assumed that there were ideal mathematical objects about which they should theorise; there were refined Platonists, called *Realists*, who worked with the notion that mathematical objects were as real as ideal physical objects about which natural philosophy reasoned, theorised and predicted with great success in the form of physics; there were those, like the Intuitionists, who worked with the hypothesis that the objects of mathematics were the creations of the human mind; and many others.

10 The first, titled: "On the Infinite", was delivered in Münster on 4 June, 1925 at a meeting organised by the Westphalen Mathematical Society to honour the memory of Karl Weierstrass, the quintessential *formalist*, The second was titled: "The Foundations of Mathematics" and delivered in July 1927 at the Hamburg Mathematical Seminar.

11 Brouwer's non-constructive fix point theorem – explicitly and forcefully rejected by him – is routinely used even in elementary economics to derive momentous normative conclusions about the feasibility of decentralised, competitive, market systems achieving efficient, computable, equilibria.

12 Hilbert's vision of the solvability of mathematical problems, and criteria for solvability, are, in my opinion, echoes of that famous Leibnizian call for settling disputes by calculation:

"What must be achieved is in fact is this: That every paralogism be recognised as an *error of calculation*, and that every *sophism* when expressed in this new kind of notation ….be corrected easily by the laws of this philosophic grammar …. Once this is done, then when a controversy arises, disputation will no more be needed between two philosophers, than *between two computers*. It will suffice that, pen in hand, they sit down .. and say to each other: *Let us calculate*."

G W Leibniz (1686/1965), p xiv; final two sets of italics added.

13 Now, Kaliningrad, named during the Soviet regime after one of the triumvirate who ruled before the Gulag years.

14 Where he was also honoured, in those enlightened pre-Nazi days, by being presented, by the Königsberg Town Council, with an "honorary citizenship".

15 Dawson (1997), p 71.

16 The marker that was placed over Hilbert's grave in Göttingen had etched on it the German original of these last two lines:

"Wir müssen wissen.
Wir werden wissen."

17 This result, in its full formal version, is known as Gödel's *Second Incompleteness Theorem*: the consistency of a mathematical system cannot be proved within that system itself.

18 Named so after the great 9th century Uzbeck mathematician from the once fertile Lake Aral region of Khorezem, Al Khowarizmi.

19 This paper by McCulloch and Pitts was highly influential in the later development of recursion theory, the mathematics of the computer. The connection between what McCulloch himself characterised as "experimental epistemology" and the foundations of mathematics was most cogently expressed by McCulloch and Pitts in this paper: "[The results of this paper afford] A *psychological* justification of the Turing definition of computability and its equivalents… .", ibid, p 35; italics added.

20 "The brain is waking and with it the mind is returning. It is as if the Milky Way entered upon some cosmic dance. Swiftly the head-mass becomes *an enchanted loom* where millions of flashing shuttles weave a dissolving pattern, always a meaningful pattern though never an abiding one; a shifting harmony of subpatterns" (Sherrington 1940: p 225; italics added).

21 Transmogrified into *The Walrasian Auctioneer*, in apologetic general equilibrium theory, in economics, entrusted with the impossible task of generating a sequence of prices that would, teleologically, lead to equilibrium configurations, as magically as Maxwell's Demon was supposed to bootleg entropy.

22 During my years as a professor at a Mexican University in the late 1990s I tried, valiantly, to find substantiation that Alexander von Humboldt's educational reform visions were decisively influenced by his meetings with Miguel Hidalgo y Costilla. Unfortunately, my own research was not deep enough to resolve the conjecture decisively.

23 Perhaps Schrödinger's main message in this slim but profound volume was summarised in the opening paragraph of the last main chapter: "What I wish to make clear …. Is, in short, that from all we have learnt about the structure of living matter, we must be prepared to find it working in a manner that cannot be reduced to the ordinary laws of physics", ibid, p 76. Such thoughts go back, also, to John Stuart Mill's "heteropathic laws", George Henry Lewis's "emergents" and C Lloyd Morgan's "emergent evolution", where non-algorithmic processes are enunciated as the basis for thought, chemical processes and the processes of the mind that give rise to consciousness. These are the ideas that form the foundations of what is fashionably referred to as the modern sciences of complexity, not negligibly founded on computability theory, too. The proverbial full circle is evident everywhere.

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